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Exploring Applications of Sequences and Series in Finance Sequences and series are fundamental mathematical concepts that have far-reaching implications in finance. While often studied in early mathematics classes, these principles hold significant power in valuation, calculation, and optimization within financial contracts. Valuation of annuity and perpetuity contracts, mortgage payments, and conversion of functions to series are some of the key applications of sequences and series in finance. Advanced mathematical concepts like Taylor's theorem enable efficient calculations using computers. This article aims to introduce sequence and series with a geometric sequence example, revisiting annuity and perpetuity contracts, and exploring their applications in calculating Present Value (PV). Series: Sum of PVs of Annuity Amounts and Perpetuity Contracts The present value (PV) of an annuity contract can be calculated using the following formula: $PV = \sum [1 - (1 + r)^{-n}]$ Let's extend this concept to perpetuity contracts, which are similar to annuity contracts but pay a fixed annual amount indefinitely. To calculate the PV of a perpetuity, we can use the same formula and replace n with infinity. Now, let's consider John, who wants to buy a home in California for \$500,000. He decides to take out a mortgage loan at an annual interest rate of 4.8%. The bank agrees to lend him the amount over 20 years, with monthly payments. To calculate the monthly payment (PMT), we can start by noting that John will make a total of 240 payments. Each payment will be discounted by the monthly interest rate of 0.004 (4.8%/12). Additionally, the sum of PVs of all these payments must equal the original principal loan amount. Using this approach, we can write an equation for the PMT: $\sum [(1 + r)^{-n}] = \text{Principal}$ To make PMT the subject of the equation, we can rearrange it as follows: $PMT = \text{Principal} / \sum [(1 + r)^{-n}]$ By using the values of the principal loan amount, monthly interest rate, and number of months, we can calculate the value of PMT to be \$3,245. This is the amount John needs to pay per month for the next 20 years to clear his mortgage loan. In conclusion, calculating the PV of an annuity or perpetuity contract involves using a formula that takes into account the present value of future cash flows. In this case, we used this concept to calculate the monthly payment required to repay a mortgage loan. A sequence is a function where the order matters, and it can be finite or infinite. The number of elements in an ordered sequence is called its length. A sequence can be defined by its domain, which is typically a countable totally ordered set like natural numbers. There are two types of sequences: infinite and finite. An infinite sequence has a domain of all natural numbers, while a finite sequence has a specific number of terms. Sequences can also be described by their general term or nth term, which generates each term in the sequence by substituting counting numbers for n. For example, if a sequence is defined as $2n-1$, the first five terms would be 1, 3, 5, 7, and 9. Similarly, if a finite sequence has four terms with a formula of $-n+1/n^2$, it produces the sequence -1, 1/2, -1/4, and 1/8. In addition to sequences, there is also the concept of series, which is the sum of a sequence. Series can be represented using summation notation, such as \sum (sigma) symbol. The Greek letter sigma (Σ) is often used to abbreviate series, allowing for concise representation of infinite sums. For example, the series $2 + 4 + 6 + 8 + 10$ with a general term $x_n = 2n$ can be written as $n=1 \sum x_n$ or $n=1 \sum 2n$. Overall, sequences and series are fundamental concepts in mathematics that provide a way to describe and analyze patterns of numbers. The formula for calculating the sum of a series is given as $\sum [k=1 \text{ to } n] f(k)$, where 'n' is the index and 1, 5 are the lower and upper limits of summation. In general, if we have a sequence x_1, x_2, \dots, x_n associated with a series, the sum can be calculated as $n \sum_{k=1}^n f_k = x_1 + x_2 + \dots + x_n$. To write out the series $\sum [k=1 \text{ to } 5] (k^2) + 1$ without using sigma notation, we calculate each term: $x_1 = 1^2 + 1 = 2$, $x_2 = 2^2 + 1 = 5$, $x_3 = 3^2 + 1 = 10$, $x_4 = 4^2 + 1 = 17$, $x_5 = 5^2 + 1 = 26$. The sum of the series is then: $2 + 5 + 10 + 17 + 26 = 60$. Similarly, to express and write the series $\sum [k=2 \text{ to } n] (-1)^k k / (k+1)$ without using sigma notation, we calculate each term for the given values of k: $x_2 = -1^2 * 2 / (2+1) = -2/3$, $x_3 = -1^3 * 3 / (3+1) = -3/4$, $x_4 = -1^4 * 4 / (4+1) = -4/5$. The sum of the series is then: $-2/3 - 3/4 - 4/5$. Now, for a sequence with general term $x_n = (-1)^n n / n$, we are asked to find the first four terms and the seventh term. The first four terms can be calculated as follows: $x_1 = (-1)^1 * 1 / 1 = -1$, $x_2 = (-1)^2 * 2 / 2 = 1$, $x_3 = (-1)^3 * 3 / 3 = -1$, $x_4 = (-1)^4 * 4 / 4 = 1$. The seventh term is then: $x_7 = (-1)^7 * 7 / 7 = -1$.

Application of sequence and series in computer science. Application of sequence and series. Sequences of services. Application of sequence and series in real life.