
I'm human

1. To find the height of the arch 6 m from the centre on either side, we can use the properties of a parabola. Let's assume the equation of the parabola is x^2 = 4ay, where 'a' is the distance from the vertex to the focus. Given that the arch is 10 m high in the centre and 30 m wide at the bottom, we can calculate the value of 'a'. Since the height at the centre is 10 m, we have a + 10 = 5 (as 'a' is the distance from the vertex to the focus). Solving for 'a', we get a = -5. However, this value doesn't make sense in the context of the problem. Let's re-examine the problem and try another approach. Since the arch is symmetrical, the height at 6 m from the centre on either side will be equal to the radius of the circle formed by the bottom of the arch. Using the Pythagorean theorem, we can find the value of 'a'. Let's assume the equation of the parabola is x^2 = 4ay. The vertex is at the origin (0, 0), and the focus is at (-5, 0). The radius of the circle formed by the bottom of the arch is half the width of the arch, which is 30 m / 2 = 15 m. Using the Pythagorean theorem, we can find the value of 'a'. a^2 + 10^2 = (30/2)^2 a^2 + 100 = 225 a^2 = 125 a = ±5√3 The height at 6 m from the centre on either side is equal to the radius of the circle formed by the bottom of the arch, which is 15 m. Therefore, the height of the arch 6 m from the centre on either side is √(15^2 - (6-(-5√3))^2) = √(225 - (6+5√3)^2) = √(225 - (36 + 60√3 + 75)) = √(54 - 60√3). 2. To find the width of the elliptical opening, we can use the equation of an ellipse. The total width of the highway is 16 m, and the height at the edge of the road must be sufficient for a truck 4 m high to clear if the highest point of the opening is to be 5 m approximately. Let's assume the equation of the ellipse is (x^2/a^2) + (y^2/b^2) = 1, where 'a' and 'b' are the semi-major and semi-minor axes respectively. The height at the edge of the road is 5 m - 4 m = 1 m. The distance from the centre to the edge of the road is half the width of the highway, which is 16 m / 2 = 8 m. Using the equation of an ellipse, we can find the value of 'a' and 'b'. (8^2/a^2) + (1^2/b^2) = 1 64/a^2 + 1/b^2 = 1 Since the height at the edge of the road is 1 m, we have b^2 = 1. Substituting this value in the above equation, we get: 64/a^2 + 1/1 = 1 64/a^2 = 0 a^2 = ∞ (not possible) However, let's try another approach. The width of the elliptical opening is given as 16 m. The ratio of the semi-major axis to the semi-minor axis is equal to the ratio of the distance from the centre to the edge of the road to the height at the edge of the road. Let's assume the equation of the ellipse is (x^2/a^2) + (y^2/b^2) = 1, where 'a' and 'b' are the semi-major and semi-minor axes respectively. The ratio of the semi-major axis to the semi-minor axis is a/b = 8/1 = 8. Substituting this value in the equation above, we get: (x^2/a^2) + (y^2/b^2) = (x^2/64) + y^2 = 1 The width of the elliptical opening is given as 16 m. The distance from the centre to the edge of the ellipse is equal to the semi-major axis, which is a = √(64*8/9) = √(512/9). Using this value in the equation above, we can find the height at the edge of the ellipse. The height at the edge of the ellipse is given by: y^2 = 1 - x^2/a^2 y^2 = 1 - (x^2/64) y = ±√(1 - x^2/64) Since the width of the elliptical opening is 16 m, we can find the value of 'b' using the equation above. The distance from the centre to the edge of the ellipse is √(512/9) = 8√3 m. The height at the edge of the ellipse is √(1 - (64/(64*8))). Therefore, the width of the elliptical opening is 2 * (√(1 - (64/(64*8)))) = 2 * (√(1 - 1/4)) = 2√3. 3. To find the value of 'b' in the equation (x^2/a^2) + (y^2/b^2) = 1, we can use the given information that the highest point of the opening is to be 5 m approximately. Let's assume the equation of the ellipse is (x^2/a^2) + (y^2/b^2) = 1, where 'a' and 'b' are the semi-major and semi-minor axes respectively. The height at the edge of the road is given as 5 m - 4 m = 1 m. The distance from the centre to the edge of the ellipse is equal to the semi-minor axis, which is 'b'. Using the equation above, we can find the value of 'a' and 'b'. The ratio of the semi-major axis to the semi-minor axis is a/b = 8/1. Substituting this value in the equation above, we get: (x^2/a^2) + (y^2/b^2) = (x^2/64) + y^2 = 1 The height at the edge of the ellipse is given by: y^2 = 1 - x^2/a^2 y^2 = 1 - (x^2/64) y = ±√(1 - x^2/64) Since the distance from the centre to the edge of the ellipse is 'b', we have b^2 = a^2/b^2. Substituting this value in the equation above, we get: a^2 = 64 a = √64 = 8 The ratio of the semi-major axis to the semi-minor axis is a/b = 8/1. The width of the elliptical opening is given as 16 m. Using this information, we can find the value of 'b'. a/b = 8/1 8 = b (not possible) However, let's try another approach. The height at the edge of the ellipse is given as √(1 - x^2/a^2). The distance from the centre to the edge of the ellipse is equal to the semi-minor axis, which is 'b'. Using the equation above, we can find the value of 'a' and 'b'. Substituting this value in the equation above, we get: a = √(64*8/9) = √(512/9) The ratio of the semi-major axis to the semi-minor axis is a/b = 8/1. The width of the elliptical opening is given as 16 m. Using this information, we can find the value of 'b'. b = a/8 b = √(512/9)/8 b = √512/72 Therefore, the value of 'a' in the equation (x^2/a^2) + (y^2/b^2) = 1 is √(512/9), and the value of 'b' is √512/72. 4. To find the height at the edge of the ellipse, we can use the equation above. The ratio of the semi-major axis to the semi-minor axis is a/b = 8/1. Substituting this value in the equation above, we get: (x^2/a^2) + (y^2/b^2) = (x^2/64) + y^2 = 1 The distance from the centre to the edge of the ellipse is √(512/9) = 8√3 m. Using this value in the equation above, we can find the height at the edge of the ellipse. The height at the edge of the ellipse is given by: y^2 = 1 - x^2/a^2 y^2 = 1 - (x^2/64) y = ±√(1 - x^2/64) However, since the width of the elliptical opening is 16 m, we can find the value of 'a' using the ratio a/b = 8/1. Substituting this value in the equation above, we get: b = √(512/9)/8 b = √512/72 Therefore, the height at the edge of the ellipse is √(1 - x^2/a^2) = √(1 - (x^2/(64))) = √((64-x^2)/64). To find the value of 'y' in this equation, we need to know the value of 'x'. Since the distance from the centre to the edge of the ellipse is 8√3 m, we can assume that x = 0. Substituting this value in the equation above, we get: y^2 = 1 - (x^2/64) y^2 = 1 y = ±1 Therefore, the height at the edge of the ellipse is y = ±1 m. Imagine throwing a ball upwards and watching it slow down as it travels higher. Using a mathematical concept called the Quadratic Equation, we can determine the position of the ball at any given time. Let's consider an example: a ball is thrown straight up from 3 meters above the ground with an initial velocity of 14 meters per second. To find out when the ball will hit the ground, we need to solve a quadratic equation that represents its height over time. The equation is h = 3 + 14t - 5t2, where h is the height and t is time in seconds. Setting the height to zero (since the ball hits the ground), we get 3 + 14t - 5t2 = 0. After simplifying and factoring, we find that the solution is t = 3, which means the ball will hit the ground after exactly 3 seconds. The graph of this equation provides further insight into the ball's trajectory, including its maximum height of nearly 13 meters. To create a new sports bicycle design, we need to consider costs and pricing strategies. With manufacturing costs of $700,000 and production costs of $110 per bike, we can use a demand curve to predict sales based on price. The demand curve is given by Unit Sales = 70,000 - 200P, where P is the price. To find the best price, we need to balance revenue with costs. By setting up equations for revenue and costs, we can determine how many bikes to produce and at what price to sell them. To find the maximum profit, we need to solve a quadratic equation. The equation is derived from the sales, costs, and profit, which are all related to the price (P). By completing the square, we can rewrite the equation as (P - 230)^2 = 10900. Taking the square root of both sides gives us P - 230 = ±104, so P = 126 or 334. However, these values represent zero profit, and we want to find the maximum profit, which occurs at a price of $230. At this price, we can expect unit sales of 24,000, sales in dollars of $5,520,000, costs of $3,340,000, and a profit of $2,180,000. This is a very profitable venture. Next, we consider an example where a company needs to cut out frames from steel sheets with a specific area of 28 cm^2. The frame's dimensions are given as 11 cm by 6 cm, and we need to find the width (x) of the metal. By solving a quadratic equation graphically, we find that x is approximately 0.8 cm. Finally, we look at an example of a river cruise where a boat travels upstream and downstream with a current speed of 2 km/h. We use algebra to solve for the boat's speed (x) in the water and find that it satisfies a quadratic equation: 3x^2 - 30x - 12 = 0. Using the quadratic formula, we can solve for x. In general, these examples demonstrate how quadratic equations can be used to model real-world problems and make informed decisions about pricing, production, and other business-related issues. By solving these equations using various methods, such as completing the square or using the quadratic formula, we can gain valuable insights into the relationships between variables and optimize outcomes. A quadratic equation problem was solved to find the speed of a boat traveling upstream or downstream. The equation resulted in two possible answers: -0.39 km/h, which doesn't make sense in this context, and 10.39 km/h, which is a perfect answer for the boat's speed. Using this speed, the time taken for the upstream and downstream journeys were calculated. The problem also explored resistors connected in parallel, where two resistors are connected with one being 3 ohms more than the other. The total resistance was measured at 2 Ohms, and using the formula RT = R1 + R2, the values of the resistors were found to be 3 ohms and 6 ohms. Additionally, the text discussed how quadratic equations are used in various fields such as optics (e.g., parabolic mirrors), architecture, and physics. Real-life examples of parabolas can be seen in everyday objects like badminton rackets, which have two parabolic shapes, and historical monuments and structures that incorporate curved lines and shapes. The text concluded by emphasizing the importance of understanding quadratic equations in real-world applications, making it a more engaging and relevant topic for students. These everyday sights can be explained using math concepts. The Eiffel Tower's bottom part, entry gates with chains on sidewalks, and bridges all follow the mathematical concept of the parabola. This shape is symmetrical around its axis and equidistant from a fixed point in the middle. Chains tied to sidewalks form parabolic figures when viewed carefully. If a tangent is drawn through the curve, it must be perpendicular to the structure. Similarly, bridges like those with tied-arch or cable-stayed shapes also depict the mathematical concept of parabola. Even fountains at amusement parks follow this shape due to their curved and symmetrical nature. Fruits like bananas can also be seen as examples of parabolas when viewed closely. Thrilling rides at amusement parks, such as roller coasters, often have tracks that form a parabolic curve. This curve is equidistant from a fixed point, creating an adrenaline rush for riders. Nature itself connects with math in surprising ways. A dolphin's jump can be seen as curved and perpendicular to its axis if a tangent is drawn, making it a classic example of a parabola. A rainbow after the rain is another natural wonder that exhibits mathematical properties. Its shape is often curvaceous and equidistant from a fixed point, showcasing the parabolic nature of a rainbow. Even everyday objects like Motorola devices have shapes that can be explained using math concepts, specifically the parabola. Parabolas are found in everyday life, from logos to food, toys, and even satellites. The M in Motorola's logo is a classic example of a parabola due to its equidistant nature. Similarly, bread loaves and swing belts exhibit this property, making them an ideal introduction to the concept. math fear. This is still an underresearched and understudied subject. The founder of Smartynote, a notepad app for dyslexia, also expresses their concern about math fear. An engineer designs a satellite dish with a parabolic cross section. The dish is 5 m wide at the opening, and the focus is placed 1.2 m from the vertex. Position a coordinate system with the origin at the vertex and the x-axis on the parabola's axis of symmetry. Find an equation of the parabola. The parabola is symmetric about the x-axis and opens to the right. The equation of the parabola is y2 = 4ax, where a = 1.2. Substitute the given point (x, 2.5) into the equation: (2.5)2 = 4a(x) 6.25 = 4.8x x = 6.25 / 4.8 x = 1.3 m Hence, the depth of the satellite dish is 1.3 m. A parabolic cable on a suspension bridge has vertical cables spaced every 6 m along its 60 m portion. Calculate the lengths of the first two vertical cables from the vertex. The equation of the parabola is (x - h)2 = 4a(y - k). Substituting the given values, we get: (x - 0)2 = 4a(y - 3) x^2 = 4a(y - 3) Substitute the point (30, 16) into the equation: 900 = 4a(13) a = 225/13 The length of the first vertical cable from the vertex is: 62 = 4(225/13)(h1 - 3) 36(13)/4(225) = (h1 - 3) h1 = 0.52 + 3 h1 = 3.52 m The length of the second vertical cable from the vertex is: 122 = 4(225/13)(h2 - 3) 144(13)/4(225) = (h2 - 3) h2 = 2.08 + 3 h2 = 5.08 m A hyperbola represents the cross-section of a nuclear cooling tower with the equation x^2/302 - y^2/442 = 1. The tower is 150 m tall, and the distance from the top to the centre of the hyperbola is half the distance from the base to the centre. The diameter of the top and base of the tower can be found by applying points A and B in the general equation.